Renormalizability and model independent description of Z' signals at low energies

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Model independent search for signals of heavy Z' gauge bosons in low-energy four-fermion processes is analyzed. It is shown that the renormalizability of the underlying theory containing Z', formulated as a scattering in the field of heavy virtual states, can be implemented in specific relations between different processes. Considering the two-Higgs-doublet model as the low-energy basis theory, the two types of Z' interactions with light particles are found to be compatible with the renormalizability. They are called the Abelian and the "chiral" couplings. Observables giving possibility to uniquely detect Z' in both cases are introduced.

I. INTRODUCTION

The existence of the heavy Z' gauge boson is predicted by a number of grand unified theories (GUT's) and superstring theories [1]. The mass of this particle is expected to be of order $m_{Z'} \geq 500$ GeV, and therefore it cannot be produced at present day accelerators. Various strategies of searching for signals of Z' as a virtual heavy state were developed and different observables convenient for its experimental detection have been introduced (see the survey [2] and references therein). The model-dependent and model-independent Z' searches at e^+e^- colliders are discussed (see, for instance, the report [3]). A popular model assumes that at low energies the Z' interactions with ordinary particles of the Standard Model (SM) can be described by the effective gauge group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \tilde{\mathrm{U}}(1)$. An alternative choice is the gauge group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ [2,4]. These models are considered as the remnants of underlying theories which are not specified. The low-energy effective Lagrangians (EL) take into consideration the most general property of renormalizable theories, ensured by the decoupling theorem [5,6] – the dominance of renormalizable interactions at low energies. The interactions of non-renormalizable types, being generated at high energies due to radiation corrections, are suppressed by the inverse heavy mass $1/m_{Z'}$. Therefore, it is possible not to consider them in leading order at lower energies. Another popular description is the introduction of the EL, considered as the sum of all effective operators with dimensions $n \geq 4$, constructed from the fields of light particles. The coefficients at these operators are treated as independent unknown numbers to be determined in experiments. For more details see Ref. [7]. In general, the number of possible Z' couplings is large. So, it is difficult to introduce observables allowing a unique detection of Z' signals. In this regard, it is desirable either to decrease the number of the independent Z' parameters on some reasonable grounds and to introduce observables most sensitive to the Z' virtual states. In any case, the main idea is to find correlations between the Z' couplings at low energies.

A straightforward way to find the correlations is to specify the underlying theory describing interactions at energies $\sim \Lambda_{\mathrm{GUT}}$ and to consider running of the couplings from high to low energies $\sim m_W$ by using the renormalization group (RG) equations. In this approach, each underlying theory leads to the unique values of the parameters and, hence, the corresponding correlations are model dependent ones. Another way is to specify a basis low-energy theory (for instance, the SM can be chosen) and to determine the relations between the Z' parameters, following from some model independent arguments. These correlations are to be model independent. Naturally, they remain dependent on the chosen basis low-energy theory.

In Refs. [8,9] the method for derivation model independent correlations between the parameters of physics beyond the SM has been developed, and new observables convenient in searching for the Z' boson in four-fermion processes were introduced. This approach is based on principles of the RG and the decoupling theorem [5]. As it was argued, any virtual heavy particle can be treated as an "external field" scattering the SM particles. The vertex describing interaction with the field contains a numeric factor, which is considered as an arbitrary parameter. Actually, it is generated by the decoupling and therefore depends on the underlying model. Due to renormalizability, the scattering amplitude in the "external field" satisfies some simple relation (named RG relation), which includes the β and γ functions entering the RG equation. These functions have to be calculated with the light particles only, and the vertex factor. Hence, relations between different vertex factors follow. Then, they can be implemented in a number of model independent observables corresponding to the specific heavy virtual state, in particular, to the Z' gauge boson [9].

In Ref. [8] as the low-energy basis model the minimal SM (with one scalar doublet) has been chosen. However, at present there is a few information about the scalar fields. In this regard, the theory with two scalar doublets is intensively studied [10,11]. The two-Higgs-

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doublet model (THDM) is also known as the low-energy limit of some E_6 based GUT's, which predict the Z' gauge boson. In the present paper, the results of Ref. [8] are generalized to the THDM case. We analyse in detail both the Abelian and the so called "chiral" types of the Z' couplings to light particles. As the latter type is concerned, it was derived as follows. We first assumed the most general parametrization of Z' interactions with the SM fields and then derived the generator structures, compatible with the renormalizability. As it will be shown in what follows, there is an important difference between these two types of interactions.

Thus, in order to derive the model independent constraints we choose the THDM as the low-energy basis theory (notice, the minimal SM is a particular case of the THDM). Then, we introduce a general parametrization of linear in Z' couplings, which is independent of the specific underlying theory. As a result, the derived RG correlations are model independent ones. They hold for the THDM as well as for the minimal SM. Moreover, the existence of other heavy particles with masses $m_i \geq m_{Z'}$ does not affect these correlations.

As it will be shown, there are two completely different sets of the Z' couplings to the SM fields compatible with renormalizability. The first one describes the Abelian Z', which respects the additional $\tilde{\mathrm{U}}(1)$ symmetry of the low energy EL. In this case the Z' couplings to the axial-vector fermion currents have a universal absolute value. The second set corresponds to the chiral Z', which interacts with the SM doublets, only. One has to distinguish these neutral Z' gauge bosons because they are described by different operators.

The content is as follows. In Sec. II the general parametrization of interactions involving the Z' and the SM fields is introduced. The RG correlations between the Z' couplings are derived in Sec. III. In Sec. IV they are compared with the specific values of the Z' couplings in the GUT's based on the E_6 group. In Sec. V the observables convenient in detection of the Z' signals are proposed. The results of our investigation are discussed in Sec. VI.

II. PARAMETRIZATION OF THE Z' COUPLINGS

In the present paper we analyze the four-fermion scattering amplitudes of order $\sim m_{Z'}^{-2}$ generated by the virtual Z' states. Vertices of interactions with more than one Z' field contribute to the amplitudes involving several virtual Z' states. The latter processes have order $m_{Z'}^{-4}$ and higher. Therefore, in what follows we consider the linear in Z' vertices, only.

To introduce a general parametrization of the vertices involving the SM fields and being linear in the Z' field, let us impose a number of natural conditions. First of all, the renormalizable type interactions are dominant at low energies $\sim m_W$. The non-renormalizable interactions generated at high energies due to radiation corrections are suppressed by the inverse heavy mass $1/m_{Z'}$ (or by other heavier scales $1/\Lambda_i \ll 1/m_{Z'}$) and therefore at low energies can be neglected in leading order. We assume that the Z' is the only neutral vector boson with the mass $\sim m_{Z'}$, and the Z' gauge field enters the theory through covariant derivatives with a corresponding charge. We also assume that the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge group of the SM is a subgroup of the GUT group. In this case, a product of generators associated with the SM subgroup is a linear combination of these generators. As a consequence, the all structure constants connecting two SM gauge bosons with Z' have to be zero. Hence, the interactions of gauge fields of the types $Z'W^+W^-$, $Z'ZZ$, and other are absent at tree level.

Let ϕ_i $(i = 1, 2)$ be two complex scalar doublets:

$$\phi_i = \left\{ a_i^+, \frac{v_i + b_i + ic_i}{\sqrt{2}} \right\}, \qquad (1)$$

where v_i marks corresponding vacuum expectation values, a_i^+ are complex fields, and b_i, c_i are real fields. By diagonalizing the quadratic terms of the scalar potential $V(\phi_1, \phi_2)$ one obtains the mass eigenstates: two neutral CP-even scalar particles, H and h, the neutral CP-odd scalar particle, A_0, the Goldstone boson partner of the Z boson, χ_3, the charged Higgs field, $H^\pm$, and the Goldstone field associated with the $W^\pm$ boson, $\chi^\pm$:

$$\begin{aligned} a_1^+ &= \chi^+ \cos\beta - H^+ \sin\beta, & a_2^+ &= H^+ \cos\beta + \chi^+ \sin\beta, \\ c_1 &= \chi_3 \cos\beta - A_0 \sin\beta, & c_2 &= A_0 \cos\beta + \chi_3 \sin\beta, \\ b_1 &= H \cos\alpha - h \sin\alpha, & b_2 &= h \cos\alpha + H \sin\alpha, \end{aligned} \qquad (2)$$

where

$$\tan\beta = \frac{v_2}{v_1}, \qquad (3)$$

and the angle α is determined by the explicit form of the potential $V(\phi_1, \phi_2)$. For instance, the CP-conserving potential, which has only CP-invariant minima, can be used [10,11]:

$$\begin{aligned} V = \sum_{i=1}^{2} \left[-\mu_i^2 \phi_i^\dagger \phi_i + \lambda_i (\phi_i^\dagger \phi_i)^2 \right] + \lambda_3 (\mathrm{Re}[\phi_1^\dagger \phi_2])^2 \\ + \lambda_4 (\mathrm{Im}[\phi_1^\dagger \phi_2])^2 + \lambda_5 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2). \end{aligned} \qquad (4)$$

It is consistent with the absence of the tree-level flavor-changing neutral currents (FCNC's) in the fermion sector. The corresponding value of α is [11]

$$\tan 2\alpha = -\frac{v_1 v_2 (\lambda_3 + \lambda_5)}{\lambda_2 v_2^2 - \lambda_1 v_1^2}. \qquad (5)$$

At low energies, when all heavy states are decoupled, the Z' interactions with the scalar doublets can be parametrized in a model independent way as follows [2]:

$$\mathcal{L}_\phi = \sum_{i=1}^{2} \left| \left(\partial_\mu - \frac{ig}{2}\sigma_a W^a_\mu - \frac{ig'}{2} Y_{\phi_i} B_\mu - \frac{i\tilde{g}}{2}\tilde{Y}_{\phi_i}\tilde{B}_\mu \right) \phi_i \right|^2 , \quad (6)$$

where g, g', $\tilde{g}$ are the charges associated with the $\mathrm{SU}(2)_L$, $\mathrm{U}(1)_Y$, and the Z' gauge groups, respectively, σ_a are the Pauli matrices,

$$\tilde{Y}_{\phi_i} = \begin{pmatrix} \tilde{Y}_{\phi_i,1} & 0 \\ 0 & \tilde{Y}_{\phi_i,2} \end{pmatrix} \quad (7)$$

is the generator corresponding to the gauge group of the Z' boson, and Y_{ϕ_i} is the $\mathrm{U}(1)_Y$ hypercharge. The condition $Y_{\phi_i} = 1$ guarantees that the vacuum is invariant with respect to the gauge group of photon.

The vector bosons, A, Z, and Z', are related with the symmetry eigenstates as follows:

$$\begin{aligned} B &\to A\cos\theta_W - (Z\cos\theta_0 - Z'\sin\theta_0)\sin\theta_W, \\ W_3 &\to A\sin\theta_W + (Z\cos\theta_0 - Z'\sin\theta_0)\cos\theta_W, \\ \tilde{B} &\to Z\sin\theta_0 + Z'\cos\theta_0, \end{aligned} \quad (8)$$

where $\tan\theta_W = g'/g$ is the adopted in the SM value of the Weinberg angle, and

$$\tan\theta_0 = \frac{\tilde{g} m_W^2 \left(\tilde{Y}_{\phi_1,2}\cos^2\beta + \tilde{Y}_{\phi_2,2}\sin^2\beta \right)}{g\cos\theta_W \left(m_{Z'}^2 - m_W^2/\cos^2\theta_W \right)}. \quad (9)$$

As is seen, the mixing angle θ_0 is of order $\sim m_W^2/m_{Z'}^2$. That results in the corrections of order $\sim m_W^2/m_{Z'}^2$ to the interactions between the SM particles. To avoid the tree-level mixing of the Z boson and the physical scalar field A_0 one has to impose the condition $\tilde{Y}_{\phi_1,2} = \tilde{Y}_{\phi_2,2} \equiv \tilde{Y}_{\phi,2}$.

Now, let us parametrize the fermion-vector interactions introducing the effective low-energy Lagrangian [2,4,12]:

$$\begin{aligned} \mathcal{L}_f = i\sum_{f_L} \bar{f}_L \gamma^\mu \Big(\partial_\mu - \frac{ig}{2}\sigma_a W^a_\mu - \frac{ig'}{2} B_\mu Y_{f_L} \\ - \frac{i\tilde{g}}{2}\tilde{B}_\mu \tilde{Y}_{f_L} \Big) f_L \\ + i\sum_{f_R} \bar{f}_R \gamma^\mu \Big(\partial_\mu - ig' B_\mu Q_f - \frac{i\tilde{g}}{2}\tilde{B}_\mu \tilde{Y}_{R,f} \Big) f_R, \end{aligned} \quad (10)$$

where the renormalizable type interactions are admitted and the summation over the all SM left-handed fermion doublets, $f_L = \{(f_u)_L, (f_d)_L\}$, and the right-handed singlets, $f_R = (f_u)_R, (f_d)_R$, is understood. Q_f denotes the charge of f in the positron charge units,

$$\tilde{Y}_{f_L} = \begin{pmatrix} \tilde{Y}_{L,f_u} & 0 \\ 0 & \tilde{Y}_{L,f_d} \end{pmatrix}, \quad (11)$$

and Y_{f_L} equals to -1 for leptons and $1/3$ for quarks.

Renormalizable interactions of fermions and scalars are described by the Yukawa Lagrangian. To avoid the existence of the tree-level FCNC's one has to ensure that at the diagonalization of the fermion mass matrix the diagonalization of the scalar-fermion couplings is automatically fulfilled. In this case the Yukawa Lagrangian, which respects the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge group, can be written in the form:

$$\begin{aligned} \mathcal{L}_{\mathrm{Yuk}} = -\sqrt{2}\sum_{f_L}\sum_{i=1}^{2} \Big\{ G_{f_d,i} \left[\bar{f}_L \phi_i (f_d)_R + (\bar{f}_d)_R \phi_i^\dagger f_L \right] \\ + G_{f_u,i} \left[\bar{f}_L \phi_i^c (f_u)_R + (\bar{f}_u)_R \phi_i^{c\dagger} f_L \right] \Big\}, \end{aligned} \quad (12)$$

where $\phi_i^c = i\sigma_2\phi_i^*$ is the charge conjugated scalar doublet, and the Cabibbo-Kobayashi-Maskawa mixing is neglected. Then, the fermion masses are

$$m_f = \frac{2m_W}{g}\left(G_{f,1}\cos\beta + G_{f,2}\sin\beta \right). \quad (13)$$

As was shown by Glashow and Weinberg [13], the tree-level FCNC's mediated by Higgs bosons are absent in case when all fermions of a given electric charge couple to no more than one Higgs doublet. This restriction leads to four different models, as discussed in Ref. [11]. In what follows, we will use the most general parametrization (12) including the models mentioned as well as other possible variations of the Yukawa sector without the tree-level FCNC's.

By using Eqs. (6), (10), and (12) it is easy to derive the Feynman rules which are collected in Appendix A.

III. RG RELATIONS

In this section we consider the correlations between the parameters $\tilde{Y}_{L,f}$, $\tilde{Y}_{R,f}$, $\tilde{Y}_{\phi_i,1}$, and $\tilde{Y}_{\phi_i,2}$ appearing due to the renormalizability of an underlying theory.

As is known, S-matrix elements are to be invariant with respect to the RG transformations, which express the independence of the location of a normalization point κ in the momentum space. In a theory with different mass scales the decoupling of heavy loop contributions at the thresholds of heavy masses, Λ, results in the important property of low energy amplitudes: the running of all functions is regulated by the loops of light particles. Therefore, the β and γ functions at low energies are determined by the SM particles, only. This fact is the consequence of the decoupling theorem [5].

Actually, the decoupling results in the redefinition of the parameters of the theory at the scale Λ and removing the all heavy particle loop contributions proportional to $\ln\kappa$ from the RG equation [6,14]:

$$\begin{aligned} \lambda_a &= \hat{\lambda}_a + a_{\lambda_a}\ln\frac{\hat{\Lambda}^2}{\kappa^2} + b_{\lambda_a}\ln^2\frac{\hat{\Lambda}^2}{\kappa^2} + ..., \\ X &= \hat{X}\left(1 + a_X\ln\frac{\hat{\Lambda}^2}{\kappa^2} + b_X\ln^2\frac{\hat{\Lambda}^2}{\kappa^2} + ... \right), \end{aligned} \quad (14)$$

where we use the notation λ_a to refer to the charges, and X represents the all fields and masses. Hats over quantities mark the parameters of the underlying theory. They include the loops of both the SM and the heavy particles, whereas the quantities without hats are calculated assuming that no heavy particles are excited inside loops. The matching between the both sets of parameters (λ_a, X and $\hat{\lambda}_a$, $\hat{X}$) is chosen to be done at the normalization point $\kappa \sim \Lambda$,

$$\lambda_a \mid_{\kappa=\Lambda} = \hat{\lambda}_a \mid_{\kappa=\Lambda}, \quad X \mid_{\kappa=\Lambda} = \hat{X} \mid_{\kappa=\Lambda} . \tag{15}$$

Since the sets of parameters λ_a, X and $\hat{\lambda}_a$, $\hat{X}$ differ at one-loop level, it is possible to substitute one set by another.

As is shown in Ref. [8], the redefinition of fields and charges (14) allows one to eliminate the one-loop mixing between heavy and light virtual states. Therefore, virtual states of heavy particles can be treated as the "external fields" scattering SM particles. The renormalizability of the underlying theory leads to some relations for vertices describing this scattering, called the RG relations.

Let us consider the four-fermion amplitudes caused by the Z' boson exchange. In the lower order in ratio $m_W^2/m_{Z'}^2$ the process $\bar{f}_1 f_1 \to Z'^* \to \bar{f}_2 f_2$ can be presented as scattering of the initial, f_1, and the final, f_2, fermions in the "external field" $1/m_{Z'}$ with the corresponding vertex factors $\Gamma_{f_1 Z'}$, $\Gamma_{f_2 Z'}$. The quantity $\Gamma_{fZ'}$ contains no contributions of heavy particle loops. Thus, it can be computed as a linear combination of the parameters $\tilde{Y}_{L,f}$, $\tilde{Y}_{R,f}$, $\tilde{Y}_{\phi_i,1}$, and $\tilde{Y}_{\phi_i,2}$.

The RG invariance of the vertex leads to equation

$$\mathcal{D}\left(\bar{f}\Gamma_{fZ'} f \frac{1}{m_{Z'}}\right) = 0, \tag{16}$$

where the effective low-energy RG operator [6] is defined as follows:

$$\mathcal{D} \equiv \frac{\partial}{\partial \ln\kappa} + \sum_a \beta_a \frac{\partial}{\partial\lambda_a} - \sum_X \gamma_X \frac{\partial}{\partial \ln X},$$
$$\beta_a = \frac{d\lambda_a}{d\ln\kappa}, \quad \gamma_X = -\frac{d\ln X}{d\ln\kappa}, \tag{17}$$

and the coefficient functions β_a and γ_X are computed taking into account the loops of light particles.

Relation (16) ensures that, as a consequence of renormalizability, the mathematical structure of the leading logarithmic terms of the vertices, calculated in one- and higher-loop approximations, coincides with that of the tree-level structures. The standard usage of Eq. (16) is to improve scattering amplitudes calculated in a fixed order of perturbation theory. In contrast, in what follows we will apply Eq. (16) to obtain an algebraic relation between the parameters $\tilde{Y}_{L,f}$, $\tilde{Y}_{R,f}$, $\tilde{Y}_{\phi_i,1}$, $\tilde{Y}_{\phi_i,2}$, which are to be considered as arbitrary numbers, since the underlying theory is not specified. Let us explain the idea in more detail. In case when the underlying theory is specified ($\tilde{Y}_{L,f}$, $\tilde{Y}_{R,f}$, $\tilde{Y}_{\phi_i,1}$, $\tilde{Y}_{\phi_i,2}$ have to be computed as discussed before), and the β and γ functions as well as the S-matrix elements are calculated in a fixed order of perturbation theory, Eq. (16) is just the identity. If the underlying theory is not specified, whereas the β, γ functions and S-matrix elements are computed in a fixed order of perturbation theory, equality (16) may serve to correlate the unknown parameters $\tilde{Y}$. In case of the four-fermion processes mediated by the gauge Z' boson, the number of independent β functions is less than the number of RG equations. Therefore, the non-trivial system of equations correlating the originally independent parameters occurs.

The one-loop RG relation for the fermion-Z' vertex is [8]

$$\bar{f}\frac{\partial\Gamma^{(1)}_{fZ'}}{\partial\ln\kappa} f \frac{1}{m_{Z'}} + \mathcal{D}^{(1)}\left(\bar{f}\Gamma^{(0)}_{fZ'} f \frac{1}{m_{Z'}}\right) = 0, \tag{18}$$

where $\Gamma^{(0)}_{fZ'}$ and $\Gamma^{(1)}_{fZ'}$ denote the tree-level and the one-loop level contributions to the fermion-Z' vertex, and $\mathcal{D}^{(1)}$ is the one-loop level part of the RG operator,

$$\mathcal{D}^{(1)} \equiv \sum_a \beta^{(1)}_a \frac{\partial}{\partial\lambda_a} - \sum_X \gamma^{(1)}_X \frac{\partial}{\partial\ln X}. \tag{19}$$

As it follows from Eq. (18), only the divergent parts of the one-loop vertices $\Gamma^{(1)}_{fZ'}$ are to be calculated. The corresponding diagrams are shown in Fig. 1. The following expressions for the right-handed and the left-handed fermions, respectively, have been obtained,

$$\begin{aligned}
\frac{\partial\Gamma^{\mu}_{f_R Z'}}{\partial\ln\kappa} = \frac{\gamma^\mu}{8\pi^2}\Big\{ & g^2 Q_f^2 \tilde{Y}_{R,f}\tan^2\theta_W + \frac{4}{3} g_{s,f}^2 \tilde{Y}_{R,f} \\
& + G_{f,1}^2\left[2T_f^3\left(\tilde{Y}_{\phi,2} + \tilde{Y}_{\phi_1,1}\right) + \tilde{Y}_{L,f} + \tilde{Y}_{L,f^\star}\right] \\
& + G_{f,2}^2\left[2T_f^3\left(\tilde{Y}_{\phi,2} + \tilde{Y}_{\phi_2,1}\right) + \tilde{Y}_{L,f} + \tilde{Y}_{L,f^\star}\right] \\
& + O\left(\frac{m_W^2}{m_{Z'}^2}\right)\Big\}, \\
\frac{\partial\Gamma^{\mu}_{f_L Z'}}{\partial\ln\kappa} = \frac{\gamma^\mu}{8\pi^2}\Big\{ & \frac{g^2}{2}\tilde{Y}_{L,f^\star} + \frac{4}{3} g_{s,f}^2 \tilde{Y}_{L,f} \\
& + g^2 \tilde{Y}_{L,f}\left[\frac{1}{4\cos^2\theta_W} + \left(Q_f^2 - |Q_f|\right)\tan^2\theta_W\right] \\
& + \left(G_{f,1}^2 + G_{f,2}^2\right)\left(\tilde{Y}_{R,f} - 2T_f^3\tilde{Y}_{\phi,2}\right) \\
& + G_{f^\star,1}^2\left(2T_f^3\tilde{Y}_{\phi_1,1} + \tilde{Y}_{R,f^\star}\right) \\
& + G_{f^\star,2}^2\left(2T_f^3\tilde{Y}_{\phi_2,1} + \tilde{Y}_{R,f^\star}\right) \\
& + O\left(\frac{m_W^2}{m_{Z'}^2}\right)\Big\},
\end{aligned} \tag{20}$$

where f and $f^\star$ are the partners of a $\mathrm{SU}(2)_L$ fermion doublet (namely, $l^\star = \nu_l$, $\nu_l^\star = l$, $q_u^\star = q_d$, and $q_d^\star = q_u$), T_f^3 is the third component of the weak isospin, and $g_{s,f}$ is the QCD charge for quarks, and zero for leptons.

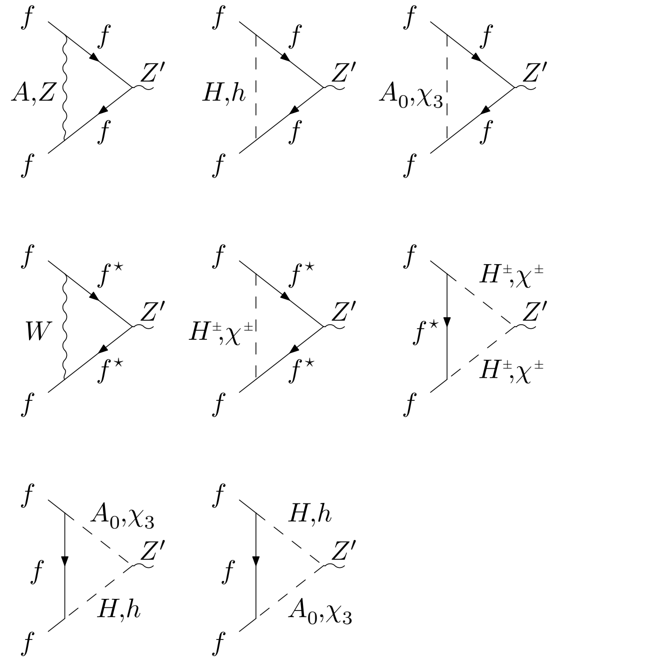

FIG. 1. One-loop contributions to the divergent part of $\Gamma_{fZ'}$.

The fermion anomalous dimensions can be calculated by using the diagrams of Fig. 2:

$$
\begin{aligned}
\gamma_{f_R} &= \frac{1}{16\pi^2}\left[g^2 Q_f^2 \tan^2\theta_W + \frac{4}{3} g_{s,f}^2 + 2\left(G_{f,1}^2 + G_{f,2}^2\right)\right. \\
&\quad \left. + O\left(\frac{m_W^2}{m_{Z'}^2}\right)\right], \\
\gamma_{f_L} &= \frac{1}{16\pi^2}\left[g^2\left(Q_f^2 - |Q_f|\right)\tan^2\theta_W + \frac{4}{3} g_{s,f}^2 + \frac{g^2}{2}\right. \\
&\quad + \frac{g^2}{4\cos^2\theta_W} + G_{f,1}^2 + G_{f,2}^2 + G_{f^\star,1}^2 + G_{f^\star,2}^2 \\
&\quad \left. + O\left(\frac{m_W^2}{m_{Z'}^2}\right)\right].
\end{aligned} \tag{21}
$$

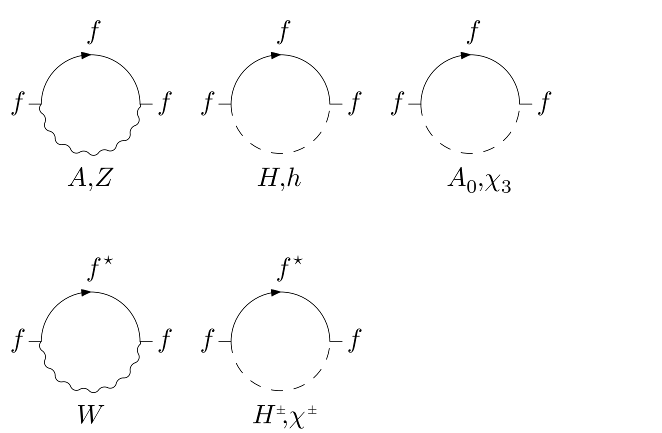

FIG. 2. One-loop contributions to the fermion mass operator.

RG relations (18) considered in a lower order in $m_W^2/m_{Z'}^2$ lead to the equations for the parameters $\tilde{Y}_{L,f}$, $\tilde{Y}_{R,f}$, $\tilde{Y}_{\phi_i,1}$, and $\tilde{Y}_{\phi_i,2}$:

$$
\begin{aligned}
&4\pi^2 \tilde{Y}_{R,f}\left(\frac{\beta_{\tilde{g}}^{(1)}}{\tilde{g}^2} + \gamma_{m_{Z'}^2}^{(1)}\right) = \\
&\quad -G_{f,1}^2\left[2T_f^3\left(\tilde{Y}_{\phi,2} + \tilde{Y}_{\phi_1,1}\right) + \tilde{Y}_{L,f} + \tilde{Y}_{L,f^\star} - 2\tilde{Y}_{R,f}\right] \\
&\quad -G_{f,2}^2\left[2T_f^3\left(\tilde{Y}_{\phi,2} + \tilde{Y}_{\phi_2,1}\right) + \tilde{Y}_{L,f} + \tilde{Y}_{L,f^\star} - 2\tilde{Y}_{R,f}\right], \\
&4\pi^2 \tilde{Y}_{L,f}\left(\frac{\beta_{\tilde{g}}^{(1)}}{\tilde{g}^2} + \gamma_{m_{Z'}^2}^{(1)}\right) = \frac{g^2}{2}\left(\tilde{Y}_{L,f} - \tilde{Y}_{L,f^\star}\right) \\
&\quad + \left(G_{f,1}^2 + G_{f,2}^2\right)\left(2T_f^3\tilde{Y}_{\phi,2} + \tilde{Y}_{L,f} - \tilde{Y}_{R,f}\right) \\
&\quad -G_{f^\star,1}^2\left(2T_f^3\tilde{Y}_{\phi_1,1} - \tilde{Y}_{L,f} + \tilde{Y}_{R,f^\star}\right) \\
&\quad -G_{f^\star,2}^2\left(2T_f^3\tilde{Y}_{\phi_2,1} - \tilde{Y}_{L,f} + \tilde{Y}_{R,f^\star}\right).
\end{aligned} \tag{22}
$$

One has to derive two sets of relations, which ensure the compatibility of Eqs. (22). The first one is

$$
\begin{aligned}
&\tilde{Y}_{\phi_2,1} = \tilde{Y}_{\phi_1,1} = -\tilde{Y}_{\phi,2} \equiv -\tilde{Y}_\phi, \\
&\tilde{Y}_{L,f} + \tilde{Y}_{L,f^\star} = 0, \quad \tilde{Y}_{R,f} = 0.
\end{aligned} \tag{23}
$$

It describes the Z' boson analogous to the third component of the $\mathrm{SU}(2)_L$ gauge field. The characteristic features of these interactions are the zero traces of generators and the absence of couplings to the right-handed singlets. In what follows, we shall call this type of interaction the "chiral" Z'. The second set,

$$
\begin{aligned}
&\tilde{Y}_{\phi_1,1} = \tilde{Y}_{\phi_2,1} = \tilde{Y}_{\phi,2} \equiv \tilde{Y}_\phi, \\
&\tilde{Y}_{L,f} = \tilde{Y}_{L,f^\star}, \quad \tilde{Y}_{R,f} = \tilde{Y}_{L,f} + 2T_f^3\tilde{Y}_\phi,
\end{aligned} \tag{24}
$$

corresponds to the Abelian Z' boson. In this case the SM Lagrangian appears to be invariant with respect to the $\tilde{\mathrm{U}}(1)$ group associated with the Z'. The first and the second relations in Eqs. (24) mean that appropriate generators are proportional to the unit matrix, whereas the third relation ensures the Yukawa terms to be invariant with respect to the $\tilde{\mathrm{U}}(1)$ transformations. Introducing the Z' couplings to the vector and the axial-vector fermion currents, $v_{Z'}^f \equiv (\tilde{Y}_{L,f} + \tilde{Y}_{R,f})/2$, $a_{Z'}^f \equiv (\tilde{Y}_{R,f} - \tilde{Y}_{L,f})/2$, one can rewrite the second and the third of Eqs. (24) in the following form:

$$
v_{Z'}^f - a_{Z'}^f = v_{Z'}^{f^\star} - a_{Z'}^{f^\star}, \quad a_{Z'}^f = T_f^3\tilde{Y}_\phi. \tag{25}
$$

As is seen, the couplings of the Abelian Z' to the axial-vector fermion currents have a universal absolute value proportional to the Z' coupling to the scalar doublets.

The solutions derived are the same as in case of the minimal SM considered in Ref. [8]. Notice that both of correlations (23) and (24) lead to the same Z' couplings to each of the scalar doublets.

Notice, in case of the Abelian Z' boson the correlations (24),(25) can be derived on related but formally different grounds. The point is that the renormalizability and gauge invariance of interactions are closely connected. Therefore, the requirement of renormalizability can be substituted by the requirement of gauge invariance of the effective low-energy Lagrangian.

In general, the EL respects by construction various [and, in particular, $\tilde{U}(1)$] symmetries. But if non-renormalizable interactions are admitted, no relations between the arbitrary parameters can be found. If only the renormalizable interactions are taken into account, as in Eq. (10), some correlations appear. In fact, to obtain formulae (24),(25) it is sufficient to require the $\tilde{U}(1)$ gauge invariance of the Yukawa terms. Note also that the correlations in Eq. (25) are the same as in the SM for the specific values of the hypercharges Y_f and Y_ϕ corresponding to the $U(1)_Y$ gauge transformations of fermion and scalar fields. On the other hand, we did not find any symmetry requirement describing the all possible relations following from Eq. (23). Therefore, the renormalizability requirement looks as more general one.

IV. RG CORRELATIONS AND THE Z' IN E_6 BASED MODELS

Over last decades the GUT's based on the E_6 gauge group [15] are intensively studied. They predict the Abelian Z' boson with the mass $m_{Z'} \gg m_W$. Since the low-energy limit of the E_6 GUT's is the THDM considered, it is of interest to check whether relations (25) hold for the specific values of the Z' couplings in these models.

There are different schemes of the E_6-symmetry breaking. One of them is

$$\begin{aligned} \mathrm{E}_6 &\to \mathrm{SO}(10)\times\mathrm{U}(1)_\psi, \\ \mathrm{SO}(10) &\to \mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times \\ &\quad \times\mathrm{U}(1)_{B-L}. \end{aligned} \tag{26}$$

This leads to the so called left-right (LR) model. Another scheme,

$$\mathrm{E}_6 \to \mathrm{SO}(10)\times\mathrm{U}(1)_\psi \to \mathrm{SU}(5)\times\mathrm{U}(1)_\chi\times\mathrm{U}(1)_\psi, \tag{27}$$

predicts the Abelian Z', which is a linear combination of the neutral vector bosons ψ and χ,

$$Z' = \chi\cos\tilde{\beta} + \psi\sin\tilde{\beta}, \tag{28}$$

where $\tilde{\beta}$ is the mixing angle.

In Table I (see Ref. [1]) we show the Z' couplings to the SM fermions in models mentioned (notice, the sign of axial-vector couplings in Ref. [1] is opposite to the sign of $a^f_{Z'}$). At first glance, some of the couplings in Table I are inconsistent with relations (25). This requires to be discussed in more detail.

First of all, let us consider the Z' couplings to neutrinos. It is usually supposed in theories based on the E_6 group that the Yukawa terms responsible for generation of the Dirac masses of neutrinos must be set to zero [15]. Therefore, there are no RG relations for the Z' interactions with the neutrino axial-vector currents, because the terms proportional to $G_{\nu,i}$ vanish in Eq. (22). In this case the couplings $a^\nu_{Z'}$ given in Table I are not restricted by relations (25).

Now, let us discuss the Z' couplings to charged leptons and quarks. The values of the couplings satisfy relations (25) in case of the LR model. As for models described by the E_6 breaking scheme (27), two possibilities of choosing $\tilde{\beta}$ are of interest. First is if the ψ boson is much heavier than the χ field. In general, this is a natural condition, since the fields ψ and χ arise at different energy scales. As a consequence, the field ψ is decoupled, and the mixing angle $\tilde{\beta}$ is small ($\tilde{\beta} \ll 1$). In this case RG relations (25) hold in lower order in $\tilde{\beta}$ for the Z' couplings to quarks and charged leptons.

The second possibility is if the masses of χ and ψ are of the same order. This means the tuning of the vacuum expectation values generating the vector boson masses. This case cannot be treated straightforwardly on the basis of relations (25) since the mixed states of the Z' bosons have to be included into consideration explicitly. Although our approach is applicable in this case, it requires additional investigation. Moreover, the Z' mixed states cause some different exchange amplitudes, which have to be incorporated into low-energy observables. In what follows, we will not discuss the case of two Z' bosons having masses of the same order.

V. OBSERVABLES

Now, let us introduce the observables convenient for detection of the Z' in electron-positron annihilation into fermion pairs $e^+e^- \to V^* \to \bar{f}f$ ($f \neq e, t$). The center-of-mass energy is taken in the range $\sqrt{s} \geq 500$ GeV. Consider the case of non-polarized initial and final fermions. Since the t quark is not considered, other fermions at these energies can be treated as massless particles, $m_f \sim 0$. In this approximation the left-handed and the right-handed fermions can be substituted by the helicity states, which will be marked as λ and ξ for the incoming electron and the outgoing fermion, respectively ($\lambda, \xi = L, R$).

Let $\mathcal{A}_V$ be the Born amplitude of the process $e^+e^- \to V^* \to \bar{f}f$ ($f \neq e, t$) with the virtual V-boson state in the s channel ($V = A, Z, Z'$). The Z' boson existence leads to the deviation of order $\sim m^{-2}_{Z'}$ of the cross section from its SM value. In general, the tree-level deviations originate from two types of contributions. The first is caused by

the Z-Z' mixing. Using the results of Sec. III the mixing angle θ_0 [see Eq. (9)] can be calculated as follows,

$$\theta_0 \simeq \frac{\tilde{g} m_W^2 \tilde{Y}_\phi}{g \cos\theta_W m_{Z'}^2}. \tag{29}$$

Because of the mixing there are corrections of order $\theta_0 \sim m_{Z'}^{-2}$ to the vertex describing interaction of Z boson and fermions. Hence, the amplitude $\mathcal{A}_Z(\theta_0)$ deviates from its SM value $\mathcal{A}_Z(\theta_0 = 0)$. The second type describes the interference between the SM amplitude, $\mathcal{A}_{\rm SM}$, and the Z' exchange amplitude, $\mathcal{A}_{Z'}$. Thus, for the process $e^+e^- \to \bar{f}f$ the deviation of the cross section is

$$\Delta\frac{d\sigma_f}{d\Omega} = \frac{d\sigma_f}{d\Omega} - \frac{d\sigma_{f,\rm SM}}{d\Omega} = \frac{\mathrm{Re}\left[\mathcal{A}^*_{\rm SM}\Delta\mathcal{A}\right]}{32\pi s} + O\left(\frac{s^2}{m_{Z'}^4}\right), \tag{30}$$

where

$$\mathcal{A}_{\rm SM} = \mathcal{A}_A + \mathcal{A}_Z|_{\theta_0=0}\,, \quad \Delta\mathcal{A} = \mathcal{A}_{Z'} + \left(\frac{d\mathcal{A}_Z}{d\theta_0}\right)_{\theta_0=0}\theta_0. \tag{31}$$

The quantity $\Delta d\sigma/d\Omega$ can be calculated in the form

$$\Delta\frac{d\sigma_f}{d\Omega} = \sum_{\lambda,\xi=L,R}\left[\mathcal{I}^{ef}_{\lambda\xi}(s) + \mathcal{M}^{ef}_{\lambda\xi}(s)\right](z + P_\lambda P_\xi)^2, \tag{32}$$

where $P_L = -1$, $P_R = 1$, $z \equiv \cos\theta$ (θ is the angle between the incoming electron and the outgoing fermion), $\mathcal{I}^{ef}_{\lambda\xi}$ denotes the Z-Z' interference term, and $\mathcal{M}^{ef}_{\lambda\xi}$ accounts of the contributions from the Z-Z' mixing:

$$\begin{aligned}
\mathcal{I}^{ef}_{\lambda\xi} &= \frac{\alpha_{\rm em}\tilde{g}^2 T_f^3 N_f}{4\pi m_{Z'}^2}\tilde{Y}_{\lambda,e}\tilde{Y}_{\xi,f}\left[|Q_f|\right. \\
&\quad \left. + \chi(s)\left(P_\lambda - \varepsilon\right)\left(P_\xi - 1 + |Q_f| - |Q_f|\varepsilon\right)\right], \\
\mathcal{M}^{ef}_{\lambda\xi} &= \frac{\alpha_{\rm em} g\tilde{g} T_f^3 N_f \theta_0}{4\pi\cos\theta_W(s - m_Z^2)}\left[\tilde{Y}_{\xi,f}\left(\delta_{\lambda,L} - 2\sin^2\theta_W\right)\right. \\
&\quad \left. + 2T_f^3\tilde{Y}_{\lambda,e}\left(2|Q_f|\sin^2\theta_W - \delta_{\xi,L}\right)\right]\left[|Q_f|\right. \\
&\quad \left. + \chi(s)\left(P_\lambda - \varepsilon\right)\left(P_\xi - 1 + |Q_f| - |Q_f|\varepsilon\right)\right],
\end{aligned} \tag{33}$$

where $\alpha_{\rm em}$ is the fine structure constant, $N_f = 3$ for quarks and $N_f = 1$ for leptons, $\varepsilon \equiv 1 - 4\sin^2\theta_W \sim 0.08$, $\chi^{-1}(s) = 16\sin^2\theta_W\cos^2\theta_W(1 - m_Z^2/s)$, and $\delta_{\lambda,\xi}$ is the Kronecker delta. The leading contribution comes from the Z-Z' interference term $\mathcal{I}^{ef}_{\lambda\xi}$, whereas the Z-Z' mixing terms are suppressed by the additional factor m_Z^2/s. At energies $\sqrt{s} \geq 500$ GeV $\mathcal{M}^{ef}_{\lambda\xi} \ll \mathcal{I}^{ef}_{\lambda\xi}$.

To take into consideration the correlations (23) or (24) let us introduce the function $\sigma_f(z)$ defined as the difference of cross sections integrated in a suitable range of $\cos\theta$ [9]:

$$\sigma_f(z) \equiv \int_z^1 \frac{d\sigma_f}{dz}dz - \int_{-1}^z \frac{d\sigma_f}{dz}dz. \tag{34}$$

The conventionally used observables – the total cross section $\sigma_{f,T}$ and the forward-backward asymmetry $A_{f,FB}$ – can be obtained by a special choice of z [$\sigma_{f,T} = \sigma_f(-1)$, $A_{f,FB} = \sigma_f(0)/\sigma_{f,T}$]. One can express $\sigma_f(z)$ in terms of $\sigma_{f,T}$ and $A_{f,FB}$:

$$\sigma_f(z) = \sigma_{f,T}\left[A_{f,FB}\left(1 - z^2\right) - \frac{1}{4}z\left(3 + z^2\right)\right]. \tag{35}$$

Then, the deviation $\Delta\sigma_f(z) \equiv \sigma_f(z) - \sigma_{f,\rm SM}(z)$ can be written in the form:

$$\begin{aligned}
\Delta\sigma_f(z) &= 4\pi\sum_{\lambda,\xi}\left[\mathcal{I}^{ef}_{\lambda\xi}(s) + \mathcal{M}^{ef}_{\lambda\xi}(s)\right] \\
&\quad \times\left(P_\lambda P_\xi - z - z^2 P_\lambda P_\xi - \frac{z^3}{3}\right).
\end{aligned} \tag{36}$$

Let us compare the observable $\Delta\sigma_f(z)$ with the differential cross section (32). As is seen, the polynomial in the polar angle z in Eq. (32) is replaced by the function of the boundary angle z in Eq. (36). The overall factor 4π appears due to the angular integration.

In what follows, we consider the observable (36) taking into account correlations (23) and (24).

A. Chiral Z'

The case of the chiral Z' corresponds to correlations (23). Because of absence of the Z' couplings to right-handed fermions the leading contribution to $\Delta\sigma_f(z)$ is proportional to the same polynomial in z for any outgoing fermion f:

$$\begin{aligned}
\Delta\sigma_f(z) &\simeq 4\pi\mathcal{I}^{ef}_{LL}(s)\left(1 - z - z^2 - \frac{z^3}{3}\right) \\
&= \frac{\alpha_{\rm em}\tilde{g}^2 T_f^3 N_f}{m_{Z'}^2}\tilde{Y}_{L,e}\tilde{Y}_{L,f}\left(1 - z - z^2 - \frac{z^3}{3}\right) \\
&\quad \times\left\{\left[|Q_f| + 2\chi(s) - |Q_f|\chi(s)\right] + O\left(\varepsilon\right)\right\}.
\end{aligned} \tag{37}$$

Therefore, the differential cross section is completely determined by the total one:

$$\begin{aligned}
\Delta\sigma_f(z) &= \Delta\sigma_{f,T}\left[\frac{3}{4}\left(1 - z - z^2 - \frac{z^3}{3}\right)\right. \\
&\quad \left. + O\left(\varepsilon, m_Z^2 s^{-1}\right)\right].
\end{aligned} \tag{38}$$

Comparing the observables for fermions of the same $\mathrm{SU}(2)_L$ isodoublet, $\{f_u, f_d\}$, it is possible to derive the correlation:

$$\Delta\sigma_{f_u}(z) = \Delta\sigma_{f_d}(z)\left[\frac{|Q_{f_u}| + 1}{|Q_{f_d}| + 1} + O\left(\varepsilon, m_Z^2 s^{-1}\right)\right]. \tag{39}$$

Hence, the ratio $\Delta\sigma_{f_u}(z)/\Delta\sigma_{f_d}(z)$ is independent of z. It equals to 5/4 for quarks and 1/2 for leptons in lower order in ε, $m_Z^2 s^{-1}$. So, the values of the observables in the $\Delta\sigma_{f_u}(z) - \Delta\sigma_{f_d}(z)$ plane are at the same curve (straight line in the approximation used) for any z specified.

It also follows from Eq. (37) that there is a value $z = z'$ when $\Delta\sigma(z') = 0$. As one can check, $z' = 2^{2/3} - 1$. Notice, the observable $\Delta\sigma(z')$ is just the variable $\Delta\sigma_-$ proposed in Ref. [16]. This quantity is completely insensitive to the chiral Z' signals. On the other hand, the deviation of the total cross section, $\Delta\sigma_T$, is more sensitive to signals of the chiral Z', since the maximum of the polynomial $1 - z - z^2 - z^3/3$ is at $z = -1$.

B. Abelian Z'

The Abelian Z' beyond the minimal SM was considered recently in Ref. [9], where sign-definite observables convenient for detection of the Abelian Z' have been introduced. RG correlations (24) in Sec. III coincide with that of Ref. [9]. Therefore, the observables for Abelian Z' beyond the THDM are to be the same as in case of the minimal SM.

In case of the chiral Z' the RG correlations (23) suppress amplitudes corresponding to the processes with right-handed fermions. This is not the case for the Abelian Z'. However, one can switch off some contributions to observable (36) by specifying the kinematic parameter z. In what follows, it will be convenient to use the Z' couplings to vector and axial-vector fermion currents [$v^f_{Z'} \equiv (\tilde{Y}_{L,f} + \tilde{Y}_{R,f})/2$, $a^f_{Z'} \equiv (\tilde{Y}_{R,f} - \tilde{Y}_{L,f})/2$]. Because of correlations (25) the absolute value of the axial-vector couplings is universal for the all types of SM fermions, $a_{Z'} \sim \tilde{Y}_\phi$. So, the observable $\Delta\sigma_f(z)$ has the form

$$\Delta\sigma_f(z) = \frac{\alpha_{\mathrm{em}}\tilde{g}^2}{m^2_{Z'}}\left[\mathcal{F}^f_0(z,s)a^2_{Z'} + \mathcal{F}^f_1(z,s)v^e_{Z'}v^f_{Z'} \right.$$
$$\left. + \mathcal{F}^f_2(z,s)a_{Z'}v^f_{Z'} + \mathcal{F}^f_3(z,s)v^e_{Z'}a_{Z'}\right]. \quad (40)$$

As it was argued in Ref. [9], one is able to choose the value of $z = z^*$, which switches off the leading contributions to the leptonic factors $\mathcal{F}^l_1$, $\mathcal{F}^l_2$, and the factor $\mathcal{F}^f_3$. The appropriate value of z^* can be found from the equation

$$\chi(s)\left(1 - z^{*2}\right) - \left(z^* + \frac{z^{*3}}{3}\right)\left[1 + \chi(s)\varepsilon^2\right] = 0. \quad (41)$$

The solution $z^*(s)$ is shown in Fig. 3. This switches off the factor at $v^e_{Z'}v^l_{Z'}$. As is seen, z^* decreases from 0.317 at $\sqrt{s} = 500$ GeV to 0.313 at $\sqrt{s} = 700$ GeV. In what follows the value of $\sqrt{s}$ is taken to be 500 GeV, because z^* and $\Delta\sigma(z)$ depend on the center-of-mass energy through the small quantity m_Z^2/s (such contributions are of order 3%).

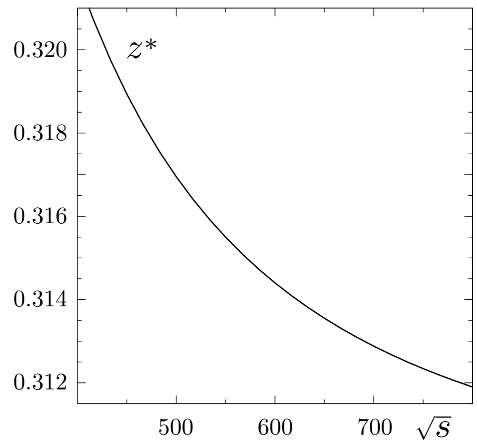

FIG. 3. z^* as the function of $\sqrt{s}$(GeV).

With the above discussed choice of z^* made, one can introduce the sign definite observable $\Delta\sigma_l(z^*)$:

$$\Delta\sigma_l(z^*) = \frac{\alpha_{\mathrm{em}}\tilde{g}^2}{m^2_{Z'}}\mathcal{F}^l_0(z^*,s)a^2_{Z'}$$
$$= -0.10\frac{\alpha_{\mathrm{em}}\tilde{g}^2\tilde{Y}^2_\phi}{m^2_{Z'}}\left[1 + O\left(0.04\right)\right] < 0. \quad (42)$$

Notice, the value of $\Delta\sigma_l(z^*)$ is universal for the all types of SM charged leptons. There are also sign definite observables for the quarks of the same generation:

$$\Delta\sigma_q(z^*) \equiv \Delta\sigma_{q_u} + 0.5\Delta\sigma_{q_d} \simeq 2.45\Delta\sigma_l\left(z^*\right) < 0. \quad (43)$$

Hence one can conclude that the values of $\Delta\sigma_{q_u}(z^*)$ and $\Delta\sigma_{q_d}(z^*)$ in the $\Delta\sigma_{q_u}(z^*) - \Delta\sigma_{q_d}(z^*)$ plane have to be at the line crossing the axes at the points $\Delta\sigma_{q_u}(z^*) = 2.45\Delta\sigma_l(z^*)$ and $\Delta\sigma_{q_d}(z^*) = 4.9\Delta\sigma_l(z^*)$, respectively.

Signals of the Abelian and the chiral Z' are compared in Figs. 4-5. Suppose for a moment that experiments give the non-zero values of leptonic observables $\Delta\sigma_l(z^*)$ ($l = \mu,\tau$). If they correspond to the Abelian Z', either of the observables has to be the same negative number. Let one also know the values of the neutrino observables $\Delta\sigma_\nu(z^*)$ ($\nu = \nu_\mu, \nu_\tau$). In case of the chiral Z' the corresponding point in Fig. 4 has to be at the straight line shown (with the accuracy of the approximation). For the Abelian Z' the shaded region as a whole is available. Now, let us consider observables for the quarks of the same generation (see Fig. 5). If the value of the leptonic observable $\Delta\sigma_l(z^*)$ is measured, one has to expect that the experimental points will be located at one of two possible curves corresponding either to the chiral or to the Abelian Z'. The shaded range represents signals of the Abelian Z' for the all possible values of the leptonic observable. So, by measuring the observables $\Delta\sigma_f(z^*)$ for fermions of the same SU$(2)_L$ isodoublet, one is able to distinguish the Abelian and the chiral Z' couplings.

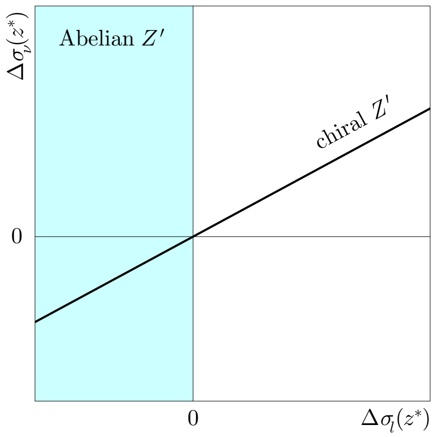

FIG. 4. Signals of the Abelian and the chiral Z' in the plane of observables $\Delta\sigma_{q_l}(z^*)$ and $\Delta\sigma_{q_{\nu_l}}(z^*)$ for leptons of the same generation. The shaded area represents the signal of the Abelian Z' for all possible values of the axial-vector couplings $a^f_{Z'}$.

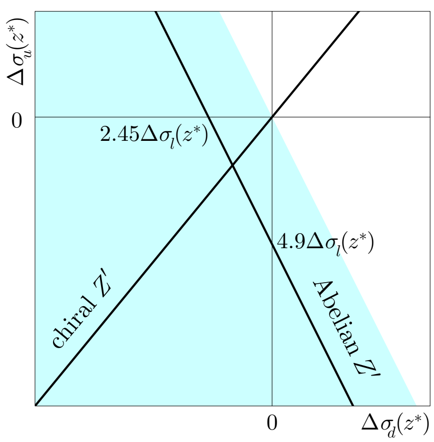

FIG. 5. Signals of the Abelian and the chiral Z' in the plane of observables $\Delta\sigma_{q_d}(z^*)$ and $\Delta\sigma_{q_u}(z^*)$ for quarks of the same generation. The shaded area represents the signal of the Abelian Z' for all possible values of the axial-vector couplings $a^f_{Z'}$.

VI. DISCUSSION

In the present paper the method of RG relations [8,17], developed originally for the minimal SM, is extended to searching for signals of the heavy Z' gauge boson beyond the THDM. General conditions when our consideration is applicable are the following. 1) The mechanism generating the heavy particle masses is not specified, and the Z' mass is treated as an arbitrary parameter. 2) The light particle masses are generated in a standard way via the non-zero vacuum values of the scalar fields of the low-energy basis theory. Interactions of light particles with heavy scalar fields, which are responsible for $m_{Z'}$, are excluded at tree level. The radiation corrections to the masses due to heavy particle loops are suppressed by factors $\sim O(m_{\text{light}}/m_{Z'})$, and therefore not taken into account. This kind of the mass hierarchy corresponds to the case when the basis theory is a subgroup of the underlying high energy model remaining unknown.

As our consideration shown, only two types of the Z' couplings to light particles are consistent with the renormalizability. The first type corresponds to the Abelian couplings respecting the $\tilde{\text{U}}(1)$ symmetry of the effective Lagrangian (10). In this case, the RG correlations fix the gauge symmetry of the Yukawa terms, which relates the fermion and the scalar hypercharges. As a consequence, the Z' couplings to the axial-vector fermion currents are completely determined by the scalar field hypercharge and the fermion isospin. The second set of solutions – chiral Z' – describes interactions with the SM particles similar to the third component of the $\text{SU}(2)_L$ gauge field. The characteristic feature of the latter couplings is the zero traces of generators associated with the Z'. Notice that the Z' interactions of the chiral type result in the effective four-fermion couplings $(\bar{f}_{1L}\gamma^\mu\sigma^a f_{1L})(\bar{f}_{2L}\gamma^\mu\sigma^a f_{2L})$ described by the operators $\mathcal{O}^{(3)}_{ll}$, $\mathcal{O}^{(3)}_{lq}$, and $\mathcal{O}^{(1,3)}_{qq}$ according to the classification in Refs. [18]. Since each type of the Z' interactions corresponds to one of mentioned operators, there is a possibility to select interactions by constructing the proper observables. As was shown, the observables proposed in Ref. [9] can be chosen in searching for the Abelian Z' boson. Thus, the bounds on the Z' couplings calculated therein are also applicable in case of the THDM.

The above note is important for the model independent search for Z' virtual states at LEP2 and future colliders LHC and NLC. In the analysis of experimental data no discriminations between these two cases have been discussed in literature (see, for instance, recent survey [1] or report [3]). This difference should be important for the model-dependent Z' search when different scenarios of symmetry breaking are discussed.

We believe that the derived RG relations to be useful in improving of experimental bounds on either the parameters of the Z' interaction with fermions and on the relations between the cross sections of various four-fermion scattering processes.

ACKNOLEDGMENTS

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APPENDIX A: FEYNMAN RULES

In what follows we use the notation $\omega_{L,R} = (1 \mp \gamma^5)/2$, and all the momenta in the vertices are understood to be incoming. The Feynman rules for vertices of Figs. 1, 2 are listed below:

1. Fermion-vector vertices

$$\bar{f} f A_\mu: \quad g \sin\theta_W Q_f \gamma^\mu;$$
$$\bar{f} f Z_\mu: \quad \frac{g}{\cos\theta_W}\gamma^\mu \left(T_f^3 \omega_L - Q_f \sin^2\theta_W\right) + O(\theta_0);$$
$$\bar{f} f Z'_\mu: \quad \frac{\tilde{g}}{2}\gamma^\mu \left(\omega_L \tilde{Y}_{L,f} + \omega_R \tilde{Y}_{R,f}\right) + O(\theta_0);$$
$$\bar{f}_d f_u W^-_\mu: \quad \frac{g}{\sqrt{2}}\gamma^\mu \omega_L;$$
$$\bar{f}_u f_d W^+_\mu: \quad \frac{g}{\sqrt{2}}\gamma^\mu \omega_L;$$

2. Fermion-scalar vertices

$$\bar{f} f H: \quad -\left(G_{f,1}\cos\alpha + G_{f,2}\sin\alpha\right);$$
$$\bar{f} f h: \quad \left(G_{f,1}\sin\alpha - G_{f,2}\cos\alpha\right);$$
$$\bar{f} f A_0: \quad 2iT_f^3\left(\omega_L - \omega_R\right) \times \left(G_{f,1}\sin\beta - G_{f,2}\cos\beta\right);$$
$$\bar{f} f \chi_3: \quad -2iT_f^3\left(\omega_L - \omega_R\right) \times \left(G_{f,1}\cos\beta + G_{f,2}\sin\beta\right);$$
$$\bar{f}_d f_u H^-: \quad \sqrt{2}\left[\omega_L\left(G_{f_d,1}\sin\beta - G_{f_d,2}\cos\beta\right) + \omega_R\left(-G_{f_u,1}\sin\beta + G_{f_u,2}\cos\beta\right)\right];$$
$$\bar{f}_u f_d H^+: \quad \sqrt{2}\left[\omega_R\left(G_{f_d,1}\sin\beta - G_{f_d,2}\cos\beta\right) + \omega_L\left(-G_{f_u,1}\sin\beta + G_{f_u,2}\cos\beta\right)\right];$$
$$\bar{f}_d f_u \chi^-: \quad \sqrt{2}\left[-\omega_L\left(G_{f_d,1}\cos\beta + G_{f_d,2}\sin\beta\right) + \omega_R\left(G_{f_u,1}\cos\beta + G_{f_u,2}\sin\beta\right)\right];$$
$$\bar{f}_u f_d \chi^+: \quad \sqrt{2}\left[-\omega_R\left(G_{f_d,1}\cos\beta + G_{f_d,2}\sin\beta\right) + \omega_L\left(G_{f_u,1}\cos\beta + G_{f_u,2}\sin\beta\right)\right];$$

3. Z' scalar vertices

$$Z'_\mu H^+ H^-: \quad \frac{\tilde{g}}{2}\left(p_{H^+} - p_{H^-}\right)_\mu \left(\tilde{Y}_{\phi_1,1}\sin^2\beta + \tilde{Y}_{\phi_2,1}\cos^2\beta\right) + O(\theta_0);$$
$$Z'_\mu H^+ \chi^-: \quad \frac{\tilde{g}\sin 2\beta}{4}\left(p_{\chi^-} - p_{H^+}\right)_\mu \times \left(\tilde{Y}_{\phi_1,1} - \tilde{Y}_{\phi_2,1}\right) + O(\theta_0);$$
$$Z'_\mu H^- \chi^+: \quad \frac{\tilde{g}\sin 2\beta}{4}\left(p_{H^-} - p_{\chi^+}\right)_\mu \times \left(\tilde{Y}_{\phi_1,1} - \tilde{Y}_{\phi_2,1}\right) + O(\theta_0);$$
$$Z'_\mu \chi^+ \chi^-: \quad \frac{\tilde{g}}{2}\left(p_{\chi^+} - p_{\chi^-}\right)_\mu \left(\tilde{Y}_{\phi_1,1}\cos^2\beta + \tilde{Y}_{\phi_2,1}\sin^2\beta\right) + O(\theta_0);$$
$$Z'_\mu H A_0: \quad \frac{i\tilde{g}}{2}\left(p_{A_0} - p_H\right)_\mu \tilde{Y}_{\phi,2}\sin(\alpha-\beta) + O(\theta_0);$$
$$Z'_\mu H \chi_3: \quad \frac{i\tilde{g}}{2}\left(p_{\chi_3} - p_H\right)_\mu \tilde{Y}_{\phi,2}\cos(\alpha-\beta) + O(\theta_0);$$
$$Z'_\mu h A_0: \quad \frac{i\tilde{g}}{2}\left(p_{A_0} - p_h\right)_\mu \tilde{Y}_{\phi,2}\cos(\alpha-\beta) + O(\theta_0);$$
$$Z'_\mu h \chi_3: \quad \frac{i\tilde{g}}{2}\left(p_h - p_{\chi_3}\right)_\mu \tilde{Y}_{\phi,2}\sin(\alpha-\beta) + O(\theta_0).$$

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TABLE I. The Z' couplings to the SM fermions in the E_6 and LR models.

f	E_6:	$a^f_{Z'}$	$v^f_{Z'}$	LR:	$a^f_{Z'}$	$v^f_{Z'}$
ν		$-3\frac{\cos\tilde{\beta}}{\sqrt{40}} - \frac{\sin\tilde{\beta}}{\sqrt{24}}$	$3\frac{\cos\tilde{\beta}}{\sqrt{40}} + \frac{\sin\tilde{\beta}}{\sqrt{24}}$		$-\frac{1}{2\alpha}$	$\frac{1}{2\alpha}$
e		$-\frac{\cos\tilde{\beta}}{\sqrt{10}} - \frac{\sin\tilde{\beta}}{\sqrt{6}}$	$2\frac{\cos\tilde{\beta}}{\sqrt{10}}$		$-\frac{\alpha}{2}$	$\frac{1}{\alpha} - \frac{\alpha}{2}$
u		$\frac{\cos\tilde{\beta}}{\sqrt{10}} - \frac{\sin\tilde{\beta}}{\sqrt{6}}$	0		$\frac{\alpha}{2}$	$-\frac{1}{3\alpha} + \frac{\alpha}{2}$
d		$-\frac{\cos\tilde{\beta}}{\sqrt{10}} - \frac{\sin\tilde{\beta}}{\sqrt{6}}$	$-2\frac{\cos\tilde{\beta}}{\sqrt{10}}$		$-\frac{\alpha}{2}$	$-\frac{1}{3\alpha} - \frac{\alpha}{2}$